Sobel 5x5 Operator for Gradient Calculation

Guennadi Levkine

Email: hlevkin@gmail.com

Mathship Technologies
Gibsons, BC, Canada.

Abstract.
One of most important tasks in Image Processing is edge detection. All algorithms for edge detection are using image gradient calculation. Prewitt and Sobel operators with 3x3 convolution kernels are usually used for that. The most popular among them is Sobel 3x3 operator. But they both give the same results. In this article we present Sobel 5x5 operator, which was developed using Local Polynomial Approximation of image brightness with special weight matrices. In some cases it is better for edge detection.
Key words: image gradient; edge detection; Prewitt operator; Sobel operator ; Local Polynomial Approximation

1 Introduction: empirical methods for image gradient calculation.

One of most important tasks in Image Processing is edge detection in images. It is based on using of gradient matrices, calculated for the image (look [4], [5])
The first suggested operator for image gradient calculation was Prewitt operator with convolution matrices for G-x and G-y components of image gradient:

-1	**0**	**1**
-1	**0**	**1**
-1	**0**	**1**

-1	**-1**	**-1**
0	**0**	**0**
1	**1**	**1**

Sobel operator was suggested in 1968 (look [2] and [3]). It has convolution matrices which were empirically introduced:

-1	**0**	**1**
-2	**0**	**2**
-1	**0**	**1**

-1	**-2**	**-1**
0	**0**	**0**
1	**2**	**1**

This operator is much more popular among developers then Prewitt. Later we compare these operators.

2 Using linear LPA for Prewitt gradient operator calculation.

In this article we use 2-dimensional Local Polynomial Approximation (LPA) method, which will be shortly described here (more detailed description is in [7]). In this method for every pixel we calculate 2D polynomial, which approximates image intensity in some block around the pixel. Here we consider only linear approximation and use special notation

$$A(x, y) = a + b*x + c*y$$

For a, b, c coefficients we use standard Least Square Fit method for minimization of difference between our approximation A(x, y) of brightness and real brightness in image.
Gradient of function *A* (*x*, *y*) is defined as a vector with coordinates:

$$G_x = \frac{\partial A}{\partial x}, \qquad G_y = \frac{\partial \mathbf{A}}{\partial \mathbf{y}}$$

In polynomial approximation the components of gradient are:

$$G_x = b\ , \qquad G_y = c$$

Using different weight matrices we can generate many different convolution kernel matrices for gradient estimation.

In this article we develop kernels for gradient calculations in 3x3 and 5x5 image blocks.
For 3x3 blocks estimation of a, b, c coefficients is obvious. Convolution kernels for their calculation are

1	**2**	**1**
2	**4**	**2**
1	**2**	**1**

-1	**0**	**1**
-1	**0**	**1**
-1	**0**	**1**

-1	**-1**	**-1**
0	**0**	**0**
1	**1**	**1**

(Weights equals 1/6, it means that we have to multiply convolution matrix onto this fraction 1/6)

If we process image with 1-byte brightness for pixel, minimal possible X and Y gradient values equal -127, maximum equals 127 (don't forget multiplier 1/6).

5x5 Kernels for gradients are (weights =1/50):

-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2

-2	-2	-2	-2	-2
-1	-1	-1	-1	-1
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2

3 Sobel 3x3 gradient operator calculation using LPA with weighted matrix.

When we calculate kernels for coefficient estimation, all the pixels in the block have the same weight. But it is possible to use different pixel weights: higher for pixels close to block center.
We consider here weight matrix (Sobel weight matrix)

1	**2**	**1**
2	**4**	**2**
1	**2**	**1**

This weight matrix is isotropic. It means that weights are dropping as 1/ *r* in any direction, where *r* is the distance between center of matrix and element of it.
Now we calculate coefficients a, b, c using this weight matrix. Kernel weights are 1/16, 1/8, 1/8):

1	2	1
2	4	2
1	2	1

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

These kernels are well known and widely used, kernel for “a” is used for noise filtration. Kernels for “b, c” called Sobel’s kernels and are used for gradient estimation.
This approach (using pixel weight matrices) gives rich opportunities for creating of many new kernels. It is convenient for solving different tasks in image processing.
For example some images have many horizontal and vertical edges. Gradients which are based on pixel weight matrix

1	3	1
3	6	3
1	3	1

are the best for edge detection in these images.
This matrix is anisotropic, because weights reduced faster in diagonal directions compare with horizon and vertical directions.
It is corresponds to Scharr gradient operators

-1	0	1
-3	0	3
-1	0	1

-1	-3	1
0	0	0
1	3	1

4. 5x5 gradient operator construction on empirical base.

Researchers are well aware of the importance of generalizing the Sobel operator to the case of a 5x5 kernel. Many different variants of this operator have been proposed, among which we have chosen to consider operator from Intel® Integrated Performance Primitives library with kernels:

$$G_x = \begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 4 & 8 & 0 & -8 & -4 \\ 6 & 12 & 0 & -12 & -6 \\ 4 & 8 & 0 & -8 & -4 \\ 1 & 2 & 0 & -2 & -1 \end{bmatrix} * A \quad \text{and} \quad G_y = \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 2 & 8 & 12 & 8 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & -8 & -12 & -8 & -2 \\ -1 & -4 & -6 & -4 & -1 \end{bmatrix} * A$$

(Look at [8])

When researchers using these kernels, there is arbitrariness in the selection of the value of the coefficient **A**. And they sometimes give wrong results for edge detection.

5. Sobel 5x5 gradient operator construction using 5x5 weighted matrix.

Let’s try to build 5x5 weight matrixes, which is generalization of 3x3 Sobel weight matrixes. In this matrix weights are decreasing by 1/ *r* law, where r is the distance between center of matrix and current element of matrix. This weight matrix looks like:

5	8	10	8	5
8	20	40	20	8
10	40	80	40	10
8	20	40	20	8
5	8	10	8	5

We call this matrix as Sobel 5x5 weight matrix. Center of it is proportional to Sobel weight matrix 3x3.
Using this weight matrix and LPA approach we found that coefficients **b, c** (gradients) are calculating using convolution matrices (weights are 1/240)

-5	-4	0	4	5
-8	-10	0	10	8
-10	-20	0	20	10
-8	-10	0	10	8
-5	-4	0	4	5

-5	-8	-10	-8	-5
-4	-10	-20	-10	-4
0	0	0	0	0
4	10	20	10	4
5	8	10	8	5

These convolution matrices have the correct coefficients for Sobel 5x5 operators.

6. Prewitt 3x3, Sobel 3x3 and 5x5 gradient operators comparison.

Gradient matrices are represented here in the color form: red color is used for pixels with positive gradient, blue color for pixels with negative gradient. It gives wider range of gradient vales representation: from -255 up to +255.
For comparison of gradient operators we use classic test image Lenna of the size 512x512 from [6]. For every gradient operator are calculated 3 corresponding 2-bytes images (2 for 3x3 gradient operators and one for 5x5 gradient operators). We analyzed only X-components of gradient.
Comparison of gradient images calculated by using 3x3 Prewitt kernel (Picture 1) and 3x3 Sobel kernel (Picture 2) demonstrate small (practically invisible) difference between them. It's because of the fact that these kernels multiplied by the right weights (1/6 and 1/8 correspondently).
In some researches Prewitt and Sobel gradient matrices are used without coefficients. Sobel gradient image is 33% brighter than Prewitt's one without that correction. That's why developers prefer Sobel operator.
Using Sobel 5x5 operator for Lenna image gradients (look Picture 3) calculation gives poor results because of loosing many edges after gradient matrix processing. For high resolution images with edges close to each other (look example at Picture 4) Sobel 5x5 operator is the best, because 3x3 gradient operators give many false edges for such type of images.

Our experiments have shown that the best way to compute gradients is:
- Find linear and non linear segments of image (look at [7] how to do it)
- Process linear parts by Prewitt 3x3 gradient kernel or Sobel 5x5 kernel,
- Use Cubic 5x5 LPA gradient operators in non linear parts of the image.

7. Conclusions.

-Prewitt 3x3 and Sobel 3x3 give the same results when use the right weights for their kernels

-They are very sensitive and gives many false edges for high resolution images

- Sobel 5x5 operator, which is built using LPA with the Sobel pixel weight matrix, gives image gradient matrices, which are much better for edge detection for high resolution images

- Modifications of pixel weight matrix allows tuning of gradient operator to groups of images with specific statistical characteristics (for example Scharr operator for images with big amount of horizon and vertical edges)

8. Acknowledgements.
I gratefully acknowledge V. Levkine for article writing support.

References:
[1] J. M. S. Prewitt. “Object Enhancement and Extraction” in “Picture Processing and Psychopictorics”, Academic Press, 1970

[2] Reference to Sobel operator in Duda, Hart “Pattern Classification and scene analysis”, Wiley & Sons, 1973

[3] Sobel I., Feldman G., "A 3x3 isotopic gradient operator for image processing", 1968, Stanford Artificial Intelligence Project

[4] Handbook of Image and Video Processing, Editor Al Bovik, Elsevier, 2005

[5] R. Gonzales, R.Woods, "Digital image processing", Pearson, 2016

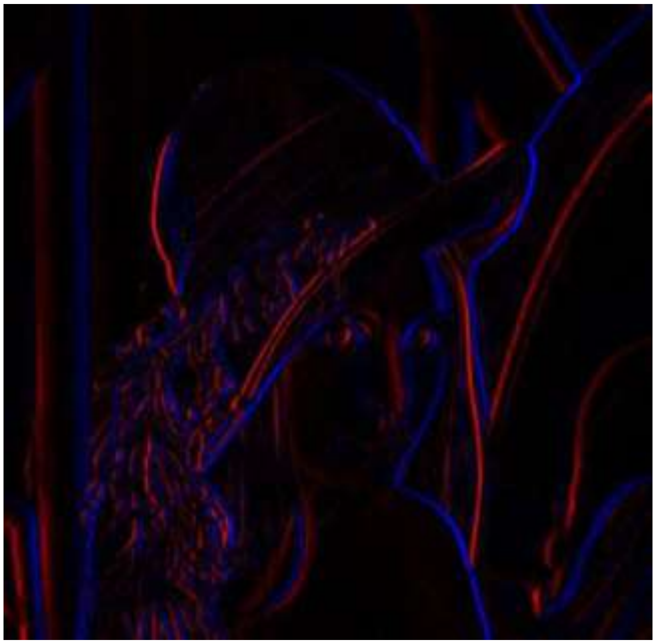
[6] http://hlevkin.com/06testimages.htm

[7] G. Levkine, “Image Local Polynomial Approximation (LPA) and its Applications”

[8] https://scc.ustc.edu.cn/zlsc/tc4600/intel/2017.0.098/ipp/common/ipp_manual/GUID-FAD741A0-8044-4DF5-B514-71F96E696D51.htm

Picture1. Prewitt 3x3 X-Gradient matrix

Picture2. Sobel 3x3 X-Gradient matrix



Picture3. Sobel 5x5 X-Gradient matrix

Picture4. 1024x1024 image Man from the Test Image Library (look at [6]).