Prewitt, Sobel and Scharr gradient 5x5 convolution matrices

Guennadi (Henry) Levkine
Email: hlevkin at yahoo.com
Vancouver, Canada.

Abstract.

Prewitt, Sobel and Scharr 3x3 gradient operators are very popular for edge detection. This article demonstrates how to get Sobel and Scharr gradient operators analytically. For that was using linear approximation of brightness in window 3x3. Every pixel in window has its own weight. Special weight matrix selection gives Sobel gradient operator. Similar considerations were made for Prewitt and Scharr gradient operators. It will also demonstrate how to build 5x5 and 7x7 Sobel and 5x5 Scharr gradient operators using similar weight matrices. Selection of different weight matrices gives a lot of new smooth and gradient operators.

Key words: image processing, image filtration, image gradient, edge detection, Prewitt operator, Sobel operator, Scharr operator, local polynomial approximation

1 Introduction

In this article we use 2-dimensional Local Polynomial Approximation (LPA) method, which will be shortly described here (more detailed description is in [1]).
In this method for every pixel we calculate 2D polynomial, which approximates image intensity in some block around the pixel. A common view for such a polynomial is

$$I(x, y) = \sum_{i,j=0}^{n} c_{ij} x^i y^j \quad (1)$$

Here we consider only linear approximation and use special notation:

$$I_1(x, y) = \alpha + \beta * x + \gamma * y \quad (2)$$

Sometimes we use quadratic approximation of brightness

$$I_2(x, y) = \alpha + \beta * x + \gamma * y + \delta * x * x + \varepsilon * x * y + \zeta * y * y \quad (2A)$$

For α, β, γ calculation we use Min Square Fit Method to minimize function

$$\Phi(\alpha, \beta, \gamma) = \sum_{i,j=1}^{i,j=3} w_{ij}{}^2 (\alpha + \beta * i + \gamma * j - I(i, j))^2 \quad (3)$$

where:

w_{ij} – Weight of pixel (i, j) in 3x3 window

α, β, γ - Coefficients of linear approximation

$I(i, j)$ – Brightness of pixel (i, j)

There are similar calculation for 5x5 window.

Gradient of function $I(x, y)$ is defined as a vector with coordinates:

$$G_x(x,y)=\frac{\partial I(x,y)}{\partial x};\ G_y(x,y)=\frac{\partial I(x,y)}{\partial y}; \tag{4}$$

In polynomial approximation the components of gradient are:

$$G_x=\beta \qquad\qquad G_y=\gamma \tag{5}$$

Using different weight matrices we can generate many different convolution kernel matrices for gradient estimation. Here we summarise masks for 3x3 and 5x5 blocks for gradient calculations.

For making of calculation more simple we give table of differences between predicted I(x,y) and measured I(x,y) in 5x5 window:

$$\begin{matrix}
(\alpha-2\beta-2\gamma-I_{-2-2}) & (\alpha-\beta-2\gamma-I_{-1-2}) & (\alpha-2\gamma-I_{0-2}) & (\alpha+\beta-2\gamma-I_{1-2}) & (\alpha+2\beta-2\gamma-I_{2-2}) \\
(\alpha-2\beta-\gamma-I_{-2-1}) & (\alpha-\beta-\gamma-I_{-1-1}) & (\alpha-\gamma-I_{0-1}) & (\alpha+\beta-\gamma-I_{1-1}) & (\alpha+2\beta-\gamma-I_{2-1}) \\
(\alpha-2\beta-I_{-20}) & (\alpha-\beta-I_{-10}) & (\alpha-I_{00}) & (\alpha+\beta-I_{10}) & (\alpha+2\beta-I_{20}) \\
(\alpha-2\beta+\gamma-I_{-21}) & (\alpha-\beta+\gamma-I_{-11}) & (\alpha+\gamma-I_{01}) & (\alpha+\beta+\gamma-I_{11}) & (\alpha+2\beta+\gamma-I_{21}) \\
(\alpha-2\beta+2\gamma-I_{-22}) & (\alpha-\beta+2\gamma-I_{-12}) & (\alpha+2\gamma-I_{02}) & (\alpha+\beta+2\gamma-I_{12}) & (\alpha+2\beta+2\gamma-I_{22})
\end{matrix}$$

For calculation of $\Phi(\alpha,\beta,\gamma)$ we make the sum of squares of elements from this table, multiplied on their weights.

2 Prewitt gradient operators 3x3, 5x5 and 7x7.

We have to select weight matrix (6) to get 3x3 Prewitt gradient operators

$$\begin{bmatrix}1 & 1 & 1\\ 1 & w & 1\\ 1 & 1 & 1\end{bmatrix} \tag{6}$$

This weight matrix is uniform- alee pixels around the center have the same weight.
For 3x3 blocks estimation of α,β,γ coefficients is obvious. Convolution kernels for their calculation are:

$$\alpha:\frac{1}{8+w^2}\begin{bmatrix}1 & 1 & 1\\ 1 & w^2 & 1\\ 1 & 1 & 1\end{bmatrix} \tag{7}$$

$$\beta:\frac{1}{6}\begin{bmatrix}-1 & 0 & 1\\ -1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix} \tag{8}$$

$$\gamma:\frac{1}{6}\begin{bmatrix}-1 & -1 & -1\\ 0 & 0 & 0\\ +1 & +1 & +1\end{bmatrix} \tag{9}$$

Convolution matrices for gradient do not depend on parameter w in the center of weight matrix (6).
If we do quantitative analysis of image, and we use gradient operators (8) and (9), for consistency we recommend to use filtration mask (7) or mask (8) from article [1]. If we process image with 1-byte brightness for pixel, minimal possible X and Y gradient values equal -127, maximum equals 127 (don't forget multiplier 1/6).

When using quadratic approximation (2A) we get the same gradient operators as (8) and (9). But for α we will get another convolution matrix

$$\alpha : \frac{1}{9}\begin{bmatrix} -1 & 2 & -1 \\ 2 & 5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

This matrix can be considered as a low pass noise filter. We call it a 3x3 Levkine noise filter (look [1]).

5x5 Prewitt weight matrix is (uniform as in the 3x3 case):

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & W & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{10}$$

For 5x5 blocks convolution kernel for calculation of α is (smoothing):

$$\alpha : \frac{1}{24+W^2}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & W^2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{11}$$

Kernels for gradients are:

$$\beta : \frac{1}{50}\begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \tag{12}$$

$$\gamma : \frac{1}{50}\begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ +1 & +1 & +1 & +1 & +1 \\ +2 & +2 & +2 & +2 & +2 \end{bmatrix} \tag{13}$$

β, γ don't depend from the parameter W in the center of weight matrix (10).

In this case (5x5 block) using quadratic LPA approximation will give the same kernels for gradient as in the case of linear approximation. Kernel for α is different and described in [1].

For the case 7x7 block coefficients α, β, γ are

$$\alpha : \frac{1}{48+W^2}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & W^2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\beta : \frac{1}{196}\begin{bmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\gamma : \frac{1}{196}\begin{bmatrix} -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

3 Sobel gradient operators 3x3, 5x5 and 7x7.

When we calculate kernels for coefficient estimation, all the pixels in the block have the same weight. But it is possible to use different pixel weights: higher for pixels close to block center. We consider here weight matrix (Sobel weight matrix)

$$\begin{bmatrix} 1/\sqrt{2} & 1 & 1/\sqrt{2} \\ 1 & w & 1 \\ 1/\sqrt{2} & 1 & 1/\sqrt{2} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & W & \sqrt{2} \\ 1 & \sqrt{2} & 1 \end{bmatrix} \qquad (14)$$

This weight matrix is isotropic. It means that weights are dropping as $1/r$ in any direction, where r is the distance between center of matrix and element of it.

Now we calculate coefficients α, β, γ using this weight matrix (14). It was found using LPA that corresponding kernels are:

$$\alpha: \ \frac{1}{12+W^2}\begin{bmatrix} 1 & 2 & 1 \\ 2 & W^2 & 2 \\ 1 & 2 & 1 \end{bmatrix} \tag{15}$$

$$\beta: \ \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{16}$$

$$\gamma: \ \frac{1}{8}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \tag{17}$$

These kernels are well known and widely used, kernel for α is used for noise filtration. Kernels for β, γ are called Sobel's kernels and are used for gradient estimation. We named kernel for α (15) as Rosenfeld's smoothing kernel.

Quadratic LPA approximation gives the same kernel for gradients (16) and (17). But for α (which can be treated as a noise filter) we get another convolution kernel:

$$\frac{1}{16}\begin{bmatrix} -1 & 2 & -1 \\ 2 & 12 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

We named this kernel as Rosenfeld - Levkine noise filter.

This approach (using pixel weight matrices) gives rich opportunities for creating kernels. It is convenient for different tasks in image processing. Let's try to build 5x5 weight matrix, which is generalization of 3x3 Sobel weight matrix. In this matrix weights are decreasing by $1/r$ law, where r is the distance between center of matrix and current element of matrix.
This weight matrix looks like:

$$\begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \\ \frac{1}{2} & 1 & w & 1 & \frac{1}{2} \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{2}} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \sqrt{5} & 2\sqrt{2} & \sqrt{10} & 2\sqrt{2} & \sqrt{5} \\ 2\sqrt{2} & 2\sqrt{5} & 2\sqrt{10} & 2\sqrt{5} & 2\sqrt{2} \\ \sqrt{10} & 2\sqrt{10} & W & 2\sqrt{10} & \sqrt{10} \\ 2\sqrt{2} & 2\sqrt{5} & 2\sqrt{10} & 2\sqrt{5} & 2\sqrt{2} \\ \sqrt{5} & 2\sqrt{2} & \sqrt{10} & 2\sqrt{2} & \sqrt{5} \end{bmatrix} \tag{18}$$

Center of it equals to Sobel weight matrix 3x3. We call this matrix as Sobel 5x5 weight matrix.
After LPA calculation we found that coefficient α calculating using convolution matrix:

$$\alpha : \frac{1}{364+W^2}\begin{bmatrix} 5 & 8 & 10 & 8 & 5 \\ 8 & 20 & 40 & 20 & 8 \\ 10 & 40 & W^2 & 40 & 10 \\ 8 & 20 & 40 & 20 & 8 \\ 5 & 8 & 10 & 8 & 5 \end{bmatrix} \tag{19}$$

Coefficients β, γ (gradients) are calculating using convolution matrices:

$$\beta : \frac{1}{240}\begin{bmatrix} -5 & -4 & 0 & 4 & 5 \\ -8 & -10 & 0 & 10 & 8 \\ -10 & -20 & 0 & 20 & 10 \\ -8 & -10 & 0 & 10 & 8 \\ -5 & -4 & 0 & 4 & 5 \end{bmatrix} \tag{20}$$

$$\gamma : \frac{1}{240}\begin{bmatrix} -5 & -8 & -10 & -8 & -5 \\ -4 & -10 & -20 & -10 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ +4 & +10 & +20 & +10 & +4 \\ +5 & +8 & +10 & +8 & +5 \end{bmatrix} \tag{21}$$

These convolution matrices (20) and (21) represent Sobel 5x5 operators for x- and y-gradients.

Quadratic approximation in 5x5 window gives the same kernels for gradients. But for coefficient α the kernel is different compared with (19).

For 7x7 windows Sobel weight matrix is

$$\begin{bmatrix} \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{13}} & \frac{1}{\sqrt{10}} & \frac{1}{3} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{13}} & \frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{13}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{13}} \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{10}} \\ \frac{1}{3} & \frac{1}{2} & 1 & \sqrt{2} & 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{13}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{5}} & \frac{1}{2} & \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{13}} \\ \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{13}} & \frac{1}{\sqrt{10}} & \frac{1}{3} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{13}} & \frac{1}{3\sqrt{2}} \end{bmatrix}$$

Corresponding kernels for α, β, γ are:

$$\alpha : \frac{1}{61692}\begin{bmatrix} 260 & 360 & 468 & 520 & 468 & 360 & 260 \\ 360 & 585 & 936 & 1170 & 936 & 585 & 360 \\ 468 & 936 & 2340 & 4680 & 2340 & 936 & 468 \\ 520 & 1170 & 4680 & 9360 & 4680 & 1170 & 420 \\ 468 & 936 & 2340 & 4680 & 2340 & 936 & 468 \\ 360 & 585 & 936 & 1170 & 936 & 585 & 360 \\ 260 & 360 & 468 & 520 & 468 & 360 & 260 \end{bmatrix}$$

$$\beta : \frac{1}{107640}\begin{bmatrix} -780 & -720 & -468 & 0 & 468 & 720 & 780 \\ -1080 & -1170 & -936 & 0 & 936 & 1170 & 1080 \\ -1404 & -1872 & -2340 & 0 & 2340 & 1872 & 1404 \\ -1560 & -2340 & -4680 & 0 & 4680 & 2340 & 1560 \\ -1404 & -1872 & -2340 & 0 & 2340 & 1872 & 1404 \\ -1080 & -1170 & -936 & 0 & 936 & 1170 & 1080 \\ -780 & -720 & -468 & 0 & 468 & 720 & 780 \end{bmatrix}$$

$$\gamma : \frac{1}{107640}\begin{bmatrix} -780 & -1080 & -1404 & -1080 & -1404 & -1080 & -780 \\ -720 & -1170 & -1872 & -2340 & -1872 & -1170 & -720 \\ -468 & -936 & -2340 & -4680 & -2340 & -936 & -468 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 468 & 936 & 2340 & 4680 & 2340 & 936 & 468 \\ 720 & 1170 & 1872 & 2340 & 1872 & 1170 & 720 \\ 780 & 1080 & 1404 & 1080 & 1404 & 1080 & 780 \end{bmatrix}$$

4. Scharr gradient operators using 3x3 and 5x5 masks

One of the popular gradient operators is Scharr operator 3x3. It also can be get from linear LPA using special weight matrix. We call it Scharr weight matrix:

$$\begin{bmatrix} \sqrt{3} & \sqrt{10} & \sqrt{3} \\ \sqrt{10} & W & \sqrt{10} \\ \sqrt{3} & \sqrt{10} & \sqrt{3} \end{bmatrix} \tag{22}$$

This matrix is not isotropic: weights in diagonal directions drop faster than in vertical and horizontal directions. After minimization of corresponding $\Phi(\alpha, \beta, \gamma)$ we get convolution matrices for α, β, γ :

$$\alpha : \frac{1}{52+w^2}\begin{bmatrix} 3 & 10 & 3 \\ 10 & w^2 & 10 \\ 3 & 10 & 3 \end{bmatrix} \quad (23)$$

$$\beta : \frac{1}{32}\begin{bmatrix} -3 & 0 & 3 \\ -10 & 0 & 10 \\ -3 & 0 & 3 \end{bmatrix} \quad (24)$$

$$\gamma : \frac{1}{32}\begin{bmatrix} -3 & -10 & -3 \\ 0 & 0 & 0 \\ +3 & +10 & +3 \end{bmatrix} \quad (25)$$

We introduce simplified Scharr like weight matrix which is also anisotropic because weights in diagonal directions drop faster compared to horizontal and vertical directions:

$$\begin{bmatrix} 1/\sqrt{3} & 1 & 1/\sqrt{3} \\ 1 & w & 1 \\ 1/\sqrt{3} & 1 & 1/\sqrt{3} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & \sqrt{3} & 1 \\ \sqrt{3} & W & \sqrt{3} \\ 1 & \sqrt{3} & 1 \end{bmatrix} \quad (26)$$

Kernels for α, β, γ calculations are:

$$\alpha : \frac{1}{16+w^2}\begin{bmatrix} 1 & 3 & 1 \\ 3 & w^2 & 3 \\ 1 & 3 & 1 \end{bmatrix} \quad (23)$$

$$\beta : \frac{1}{10}\begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -1 & 0 & 1 \end{bmatrix} \quad (24)$$

$$\gamma : \frac{1}{10}\begin{bmatrix} -1 & -3 & -1 \\ 0 & 0 & 0 \\ 1 & 3 & 1 \end{bmatrix} \quad (25)$$

Comparing with Sobel operators we see that they look similar, but Sobel weight matrix is isotropic and Scharr weight matrix is anisotropic, where diagonal elements have less weights comparing with Sobel diagonal weights.

Here is generalization of simplified Scharr weight matrix to 5x5 case:

$$\begin{bmatrix} 1/(2\sqrt{3}) & 1/\sqrt{6} & 1/2 & 1/\sqrt{6} & 1/(2\sqrt{3}) \\ 1/\sqrt{6} & 1/\sqrt{3} & 1 & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/2 & 1 & w & 1 & 1/2 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1 & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/(2\sqrt{3}) & 1/\sqrt{6} & 1/2 & 1/\sqrt{6} & 1/(2\sqrt{3}) \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \sqrt{2} & 2 & \sqrt{6} & 2 & \sqrt{2} \\ 2 & 2\sqrt{2} & 2\sqrt{6} & 2\sqrt{2} & 2 \\ \sqrt{6} & 2\sqrt{6} & W & 2\sqrt{6} & \sqrt{6} \\ 2 & 2\sqrt{2} & 2\sqrt{6} & 2\sqrt{2} & 2 \\ \sqrt{2} & 2 & \sqrt{6} & 2 & \sqrt{2} \end{bmatrix} \tag{26}$$

We found this matrix, based on assumptions:

- In horizontal and vertical directions the weight is proportional to reciprocal distance from the center
- In diagonal directions the weight is proportional to (reciprocal distance)* $\sqrt{2/3}$
- It the direction between horizontal and diagonal we use assumption that the weight is proportional to (reciprocal distance)* $\sqrt{5/6}$

After calculation we found that α, β, γ equals:

$$\alpha : \frac{1}{84+W^2}\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 12 & 4 & 2 \\ 3 & 12 & W^2 & 12 & 3 \\ 2 & 4 & 12 & 4 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix} \tag{27}$$

$$\beta : \frac{1}{60}\begin{bmatrix} -1 & -1 & 0 & 1 & 1 \\ -2 & -2 & 0 & 2 & 2 \\ -3 & -6 & 0 & 6 & 3 \\ -2 & -2 & 0 & 2 & 2 \\ -1 & -1 & 0 & 1 & 1 \end{bmatrix} \tag{28}$$

$$\gamma : \frac{1}{60}\begin{bmatrix} -1 & -2 & -3 & -2 & -1 \\ -1 & -2 & -6 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ +1 & +2 & +6 & +2 & +1 \\ +1 & +2 & +3 & +2 & +1 \end{bmatrix} \tag{29}$$

It will be interesting to develop Scharr weight matrix 7x7, which is consistent with 3x3 and 5x5 weight matrices.

5 New 5x5 convolution matrices (modified Prewitt) for gradient calculation

Weight matrix here is:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & w & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 1 & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 1 \\ 1 & 2 & 2*w & 2 & 1 \\ 1 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad (30)$$

Convolution matrix for coefficient α is:

$$\alpha : \frac{1}{48+4*w^2} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 4 & 4 & 4 & 1 \\ 1 & 4 & 4*w^2 & 4 & 1 \\ 1 & 4 & 4 & 4 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad (31)$$

Convolution matrices for gradient are:

$$\beta : \frac{1}{68} \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -4 & 0 & 4 & 2 \\ -2 & -4 & 0 & 4 & 2 \\ -2 & -4 & 0 & 4 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix} \qquad (32)$$

$$\gamma : \frac{1}{68} \begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ -1 & -4 & -4 & -4 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ +1 & +4 & +4 & +4 & +1 \\ +2 & +2 & +2 & +2 & +2 \end{bmatrix} \qquad (33)$$

These convolution matrices need to be investigated for possible uses.

6 Comparing of Prewitt, Sobel and Scharr 3x3 and 5x5 gradient operators.

For comparison of gradient operators we use classic test image Lenna of the size 512x512 from [2]. For every gradient operator we calculate 6 corresponding 2-bytes image (3 for 3x3 gradient operators and another 3 for 5x5 gradient operators). They are presented in Pic.1 – Pic.6. After that we compare results for 3x3 Prewitt, Sobel and Scharr and results for 5x5 Prewitt, Sobel and Scharr.

We are working with grey scale Lenna image, but gradient matrices are represented in the color form: red color is used for pixels with positive gradient, blue color for pixels with negative gradient.

Pic.1, Pic.2 and Pic.3 show us small (practically invisible) difference between them. It's because of the fact, that Lenna image can be presented in 3x3 windows by quadric surfaces and 3x3 Prewitt, Sobel, Scharr gradients gives right results in this case.
In some articles Prewitt and Sobel gradient matrices are used without coefficients (1/6 and 1/8 correspondently). As a result they had Sobel gradient image 33% brighter than Prewitt's one. Similar results will happen when using Scharr gradient operator without coefficient (1/32).

For 5x5 gradient operators the situation is different. Lenna image is not good for 5x5 quadric surfaces presentation. As a result 5x5 Prewitt operator will give us not good but smoothed gradient image. Sobel and Scharr gives better results because of the fact that far pixels in 5x5 window have less weights then pixels close to the center. 5x5 Sobel and Scharr gradient images are practically the same.

Some images have many horizontal and vertical edges. In addition to the 1-dimensional gradient operators it will be useful to use the Scharr gradient operators because of its anisotropic nature.

7. Conclusions.

LPA together with weight matrix selection is useful for building of smoothing and gradient operators for image processing. For every gradient operator we have set of smoothing operators, which using makes consistency in simultaneous use of smoothing and gradient operators.
For big images, it is often convenient to use 5x5 and 7x7 convolution matrices.
It will be interesting to use other weight matrices, which build by using other distance metrics (look [12]).

8. Acknowledgements.

I gratefully acknowledge V. Levkine for the support in writing this article.
I thank I. Ragnemalm for sending me Sobel's letter (look [11]).

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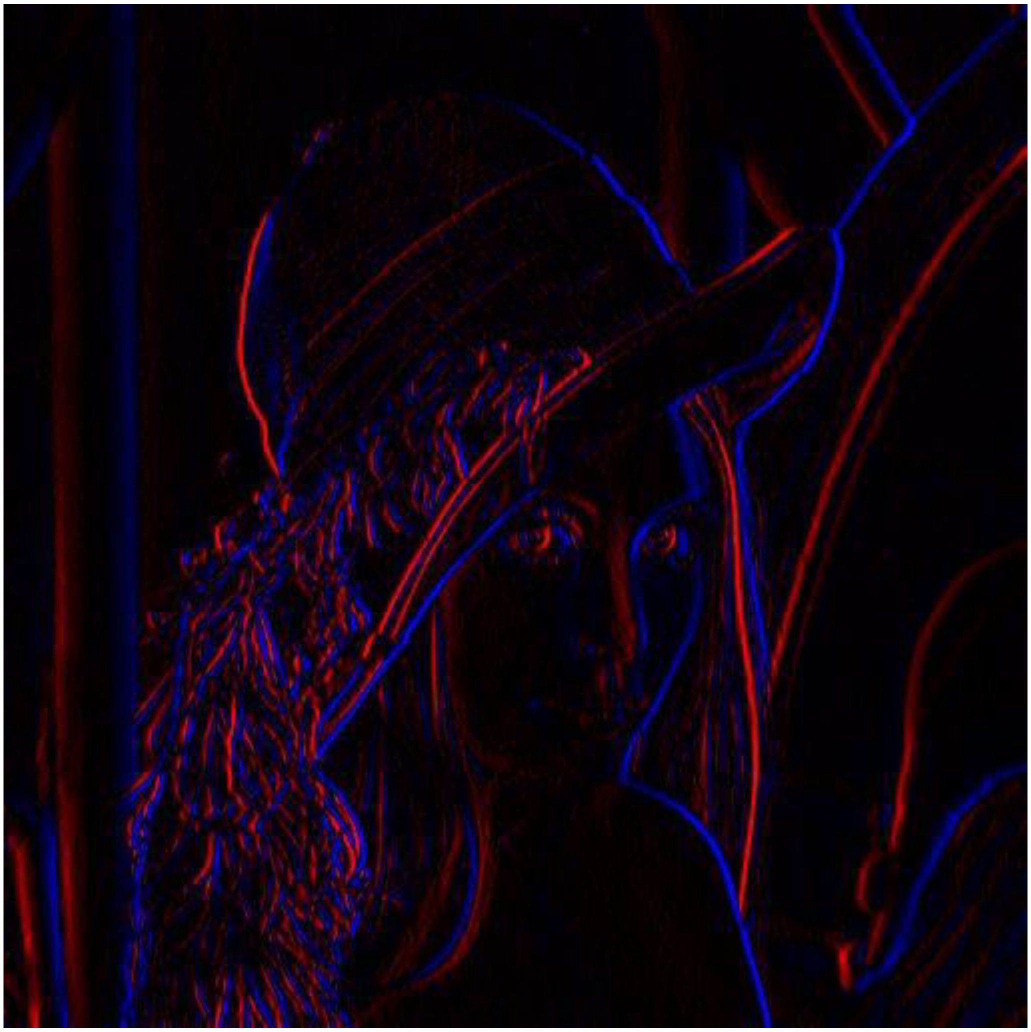
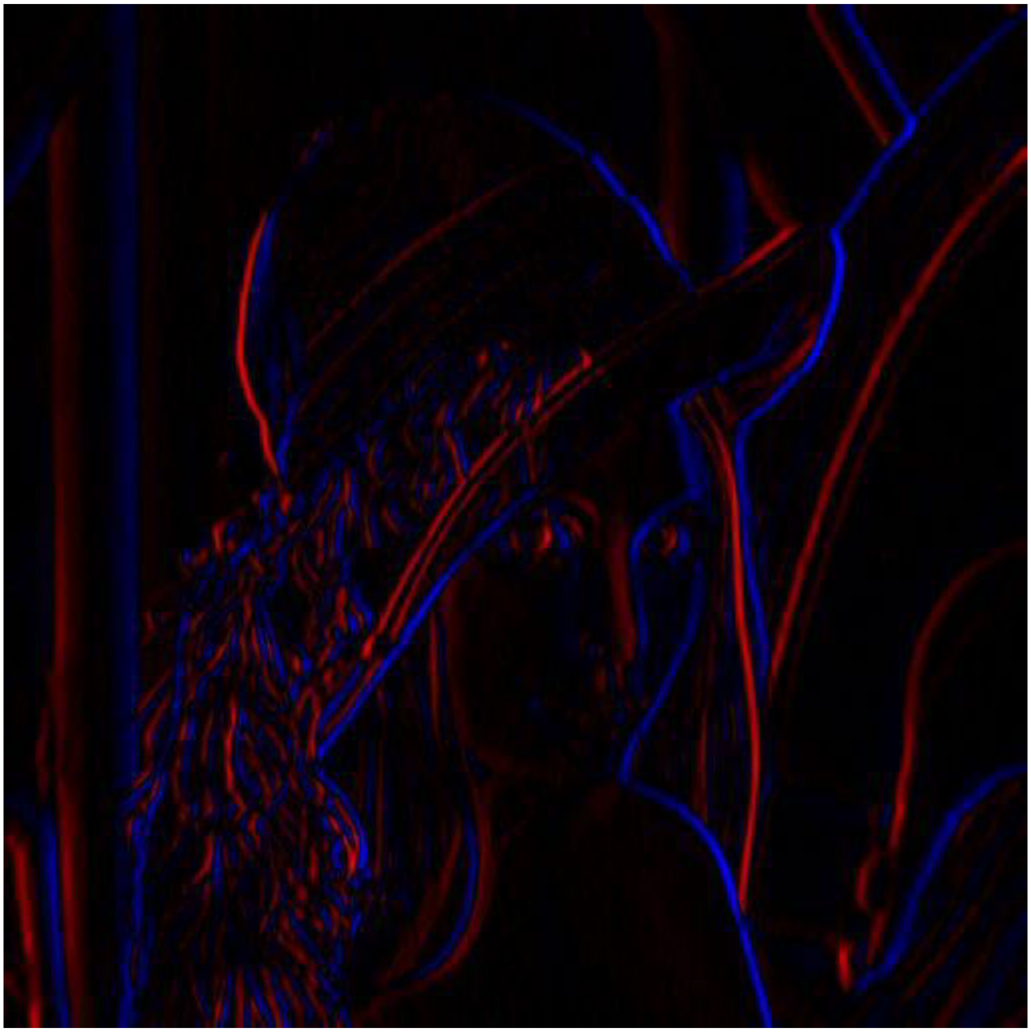


Pic.1 Prewitt 3x3 horizon gradient of Lenna

Pic.2 Sobel 3x3 horizon gradient of Lenna

Pic.3 Simplified Scharr 3x3 horizon gradient of Lenna



Pic.4 Prewitt 5x5 horizon gradient of Lenna

Pic.5 Sobel 5x5 horizon gradient of Lenna

Pic.6 Simplified Scharr 5x5 horizon gradient of Lenna